

L211 Logic and Mathematics

4. Lecture

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Last lecture

Proofs

IMPORTANT POINTS

- ▶ $\wedge, \vee, \rightarrow, \neg, \forall, \exists$
- ▶ $\neg\forall nP(n) \Leftrightarrow \exists n\neg P(n)$
- ▶ necessary, sufficient, necessary and sufficient
- ▶ truth tables
- ▶ direct proof method
- ▶ contraposition: $A \rightarrow B \Leftrightarrow \neg B \rightarrow \neg A$
- ▶ proof by contradiction

Induction

PRINCIPLE OF INDUCTION

Considering a proposition $P(n)$

Show that $P(0)$ hold

and

show that for all natural numbers n , $P(n) \rightarrow P(n+1)$ holds.

Then

$P(n)$ holds for all natural numbers.

RULE OF INDUCTION

$$\frac{P(0) \quad \forall n \in \mathbb{N} : P(n) \rightarrow P(n+1)}{\forall k \in \mathbb{N} : P(k)}$$

Example

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

CONCLUSION 1

- ▶ write down that you use induction
- ▶ write down what is the sentence you want to prove
- ▶ prove $P(0)$
- ▶ prove $P(n) \rightarrow P(n+1)$
- ▶ qed \square

Example

$$\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}$$

Example

$$3|(n^3 - n)$$

CONCLUSION 2

One needs to use the assumption

Example

Everyone loves Funazushi!

FUNAZUSHI

Proposition $P(n)$

Within a group of n people, everyone
has
the same feeling regarding Funazushi.

Conclusion

I like Funazushi
thus everyone likes Funazushi.

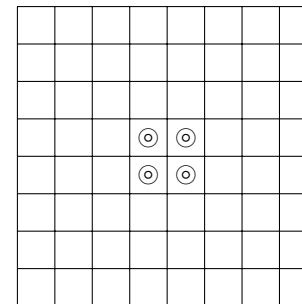
CONCLUSION 3

Don't get tricked! It is not about convincing each other!

Example

The center of a stone garden

LET'S PLASTER A $2^n \times 2^n$ STONE GARDEN
WITH TILES



Tile shape



All $2^n \times 2^n$ stone gardens can be tiled, so that in the center one
piece remains.

CONCLUSION 4

Sometimes it is better to use a stronger induction hypothesis!

Next lesson

More induction