L211 Logic and Mathematics

2. Lecture

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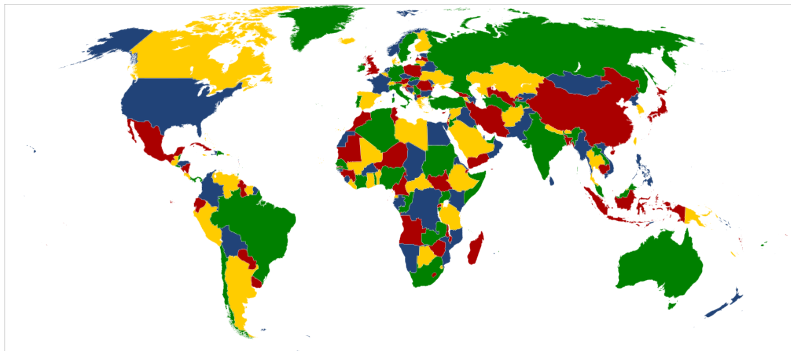
$x^n + y^n = z^n$

Last weeks lecture

Mathematics in the 20[th] century



The sentence in the frame of this slide is wrong

Today's lecture

From high school math to university math

From calculation to a science of patterns

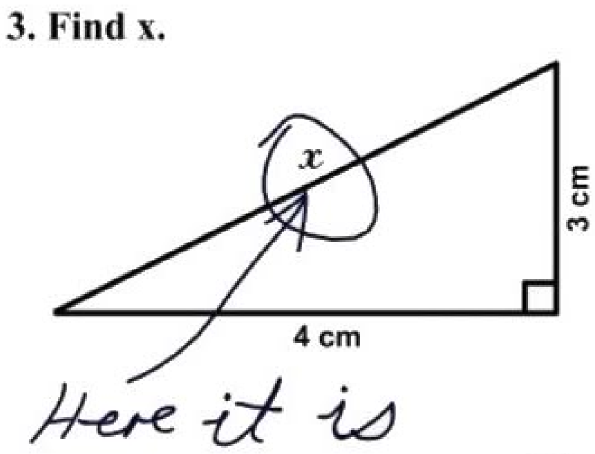
School math

Calculation

find x

$$x^2 + x - 2 = 0$$

Trigonometry and geometry



University math

Probability theory

What is the probability?

Diophantine equations

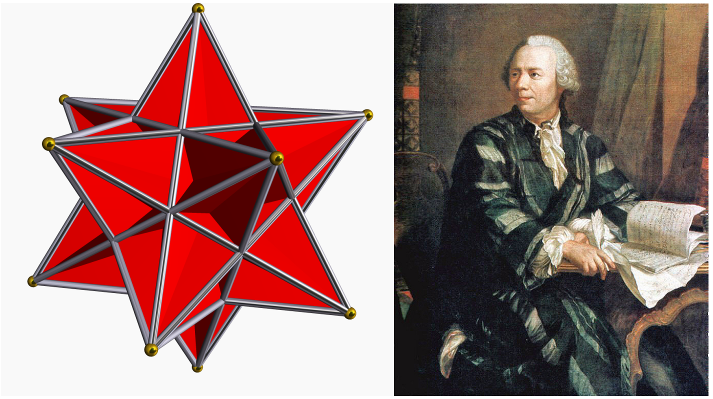
Linear equation

$$ax + by = c \quad \text{such that } a, b, c \text{ are integers } (\in \mathbb{Z})$$

What are the conditions for a, b, c?

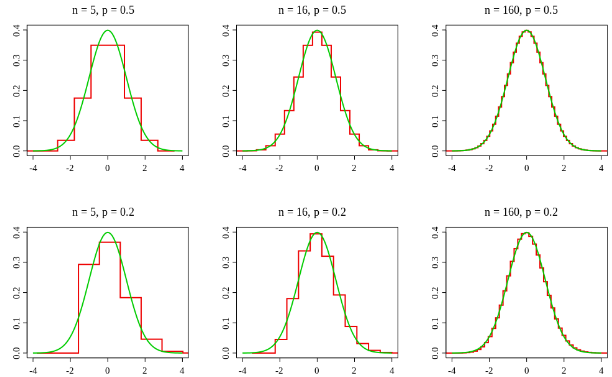
– c needs to be a divisor of the gcd of a and b

Euler's Polyhedra formula



$$V - E + F = 2$$

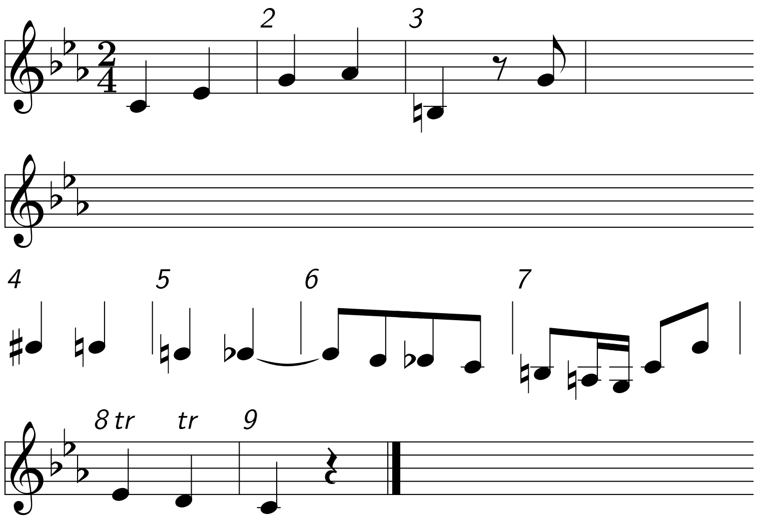
Central Limit Theorem



$$P\left(\frac{S_n - n\mu}{\sqrt{n}\sigma} \leq \alpha\right) \to \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha} e^{-\frac{x^2}{2}}\, dx$$

Patterns

Notes and Music



$$
\begin{aligned}
|Q(t_1, T)| &\leq \frac{1}{\epsilon_0} \int_{t_1}^{T} \int_M e^{\psi} H \\
&= \frac{1}{\epsilon_0} \{ |M(t_1)| - |M(T)| \} \\
&\leq \frac{1}{\epsilon_0} |M(t_1)|.
\end{aligned}
$$

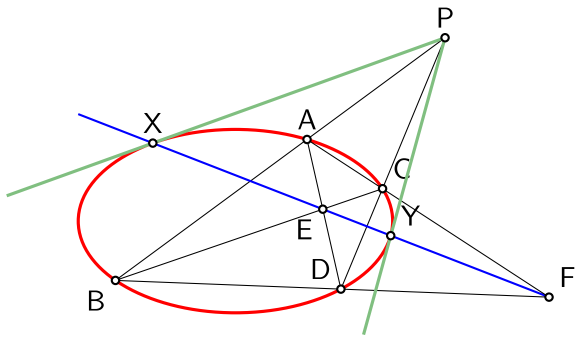
Theory of Patterns

$$n + m = m + n$$

$$\Downarrow$$

pattern of numbers/counting/calculation

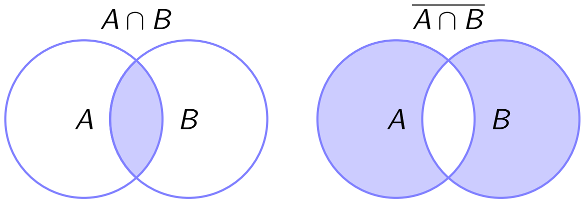
arithmetic



$$\Downarrow$$

pattern of shapes and forms

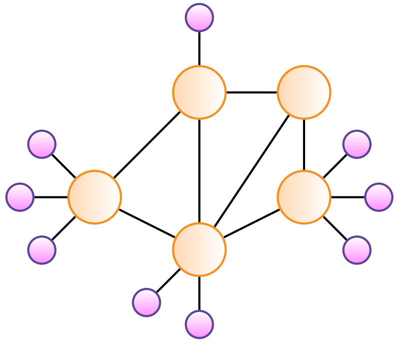
geometry



$\Downarrow$

pattern of dispute/discussion

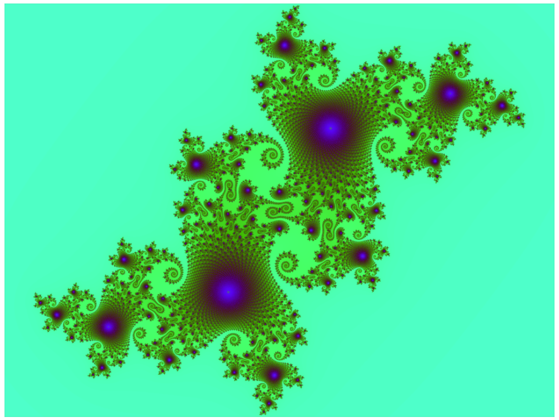
logics



$\Downarrow$

pattern of position

topology



$\Downarrow$

pattern of dynamics

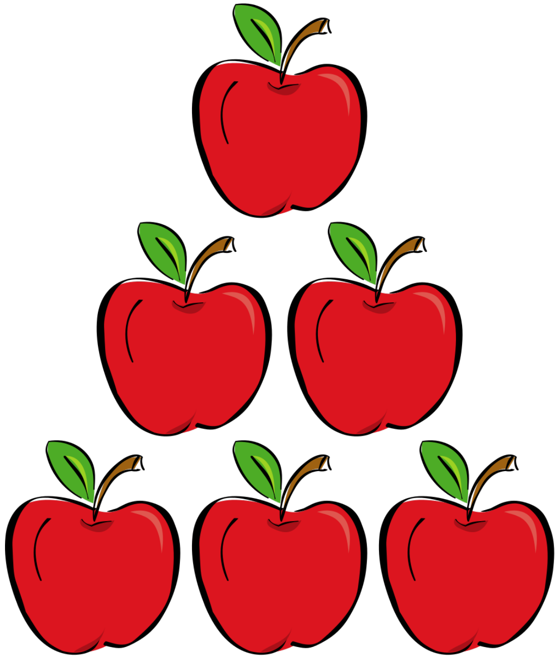
Recapitulation

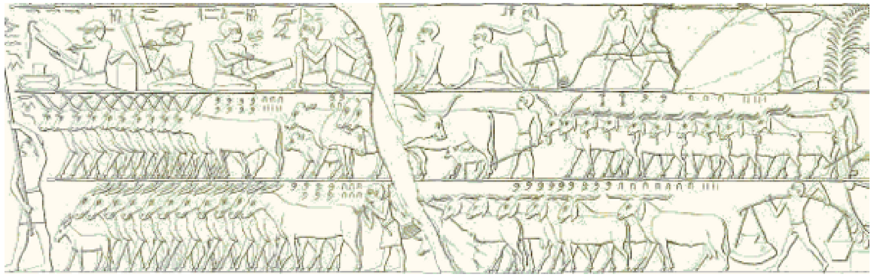
School	calculation – search for solution
University	abstraction and generalization

First generalization

From natural numbers to reals

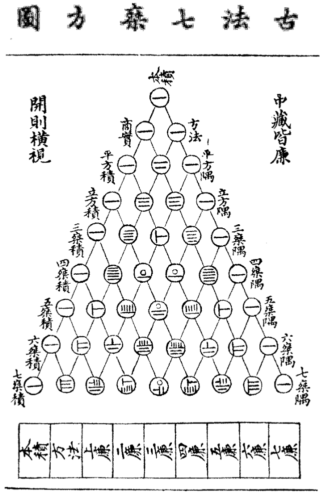
First step



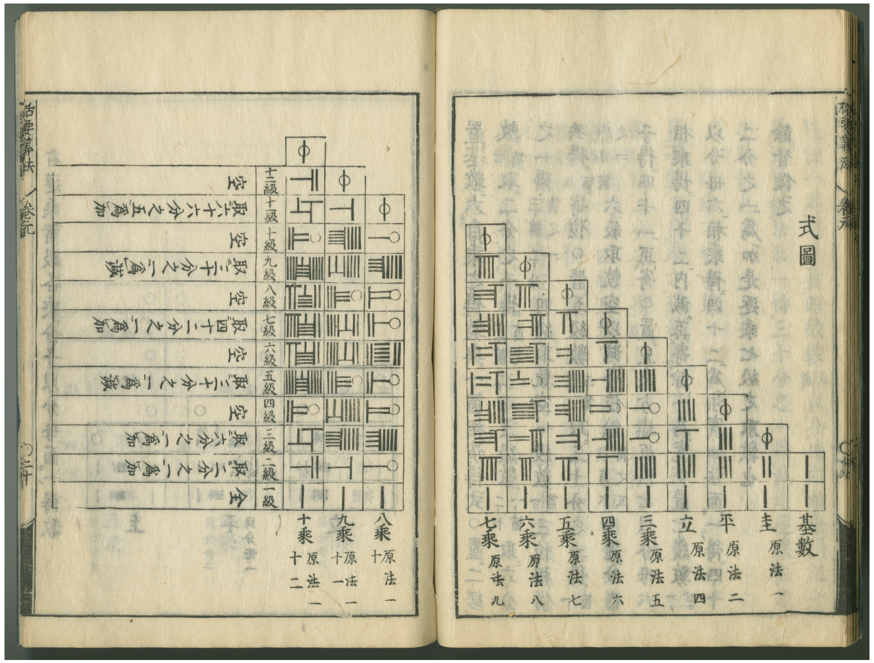


Cattle Count, Ancient Egypt. From Lepsius Denkmahler.

King's Son Wepemnefret. Phoebe A. Hearst Museum of Anthropology. Photograph by Bruce White.



Yang Hui () (Pascal's) triangle, as depicted by Zhu Shijie in 1303, using rod numerals.



finger couting

- finger counting
- touchable objects

first generalization

natural number — any set of objects of the same count

6 =the set of all 'things' of 6 elements

From now on, let us take 6 as simply 6 ;-)

Leopold Kronecker

- 1823–1891
- German mathematician

$\boxed{\text{The integers are made by God, everything else by humans}}$

Die ganzen Zahlen hat der liebe Gott gemacht, alles andere ist Menschenwerk.

Operations and functions on (positive) natural numbers

- addition
- multiplication
- order

prime numbers

any number that is only divisible by 1 and itself

Fundamental theorem of arithmetic

Every natural number has exactly
one representation as product of primes

Prime number factorization

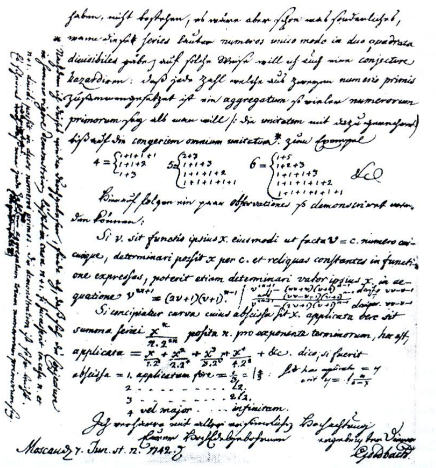
Sieve of Eratosthenes

factorization of big numbers is difficult – public key encryption

- Number of primes? infinite – proof? – homework
- Prime number theorem
  What is the rate of prime numbers?

$$\pi(n) \sim \frac{n}{\ln n}$$

n	$\pi(n)$	$\frac{n}{\ln n}$	$\frac{\pi(n)}{x/\ln x}$
10	4	4.34...	0.921...
100	25	21.71...	1.151...
10000	1229	1085.73...	1.1320...
100000	9592	8685.89...	1.1043...
1000000	78498	72382.41...	1.0845...
10000000	664579	620420.69...	1.0712...
100000000	5761455	5428681.02...	1.0613...



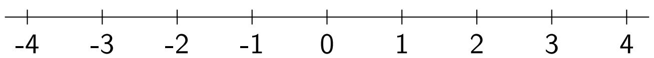
- Goldbach Conjecture

Every even number bigger than 2 can be expressed as the sum of two primes.

true for $n \leq 4 \times 10^{18}$

From positive to negative numbers

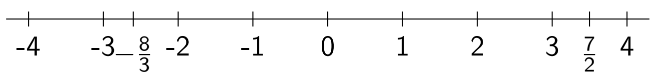
- Addition is total
  For any a and b, the addition $a + b$ is defined.
- What about subtraction?



- Completion with respect to subtraction: $\mathbb{N} \to \mathbb{Z}$

From Integers to Rational Number

- Distribution of a birthday cake into 5 pieces
- Current state: For all integers a, b
  $a + b$, $a - b$, $a \times b$ are defined.



- Completion with respect to division: $\mathbb{Z} \to \mathbb{Q}$

Calculation with Rationals/Fractions

Addition $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

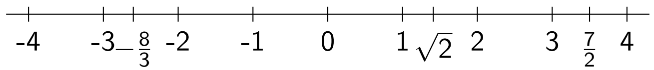
Multiplication $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

Division $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

From Rationals to the Reals

- $\sqrt{2}$?
- Current status: For all rationals a, b
  $a+b$, $a-b$, $a \times b$, $a \div b$ is defined.



- For any real number r – approximation with rational numbers is possible
- decimal notation system

A bit irrational and very irrational

- $a_n x^n + b_{n-1} x^{n-1} + \cdots + a_0 = 0$ with $a_k \in \mathbb{Q}$
- algebraic numbers – solutions of polynoms with rational coefficients $\mathbb{A}$
- completion with respect to rational polynoms: $\mathbb{Q} \rightarrow \mathbb{A}$

Are we finished here?

Transcendental numbers

- π, e, ...

From the reals to the complex numbers

- $\sqrt{-1}$?
- Current status: for all real numbers a, b $a + b$, $a - b$, $a \times b$, $a \div b$ is defined.
- completion with respect to solving polynomial equations: $\mathbb{R} \to \mathbb{C}$
- And after that? Quaternions

Recapitulation

- School Mathematics and University Mathematics are different
- Theory of patterns
- Discovery of patterns of numbers
- Abstraction and generalization $\mathbb{N} \to \mathbb{Z} \to \mathbb{Q} \to \mathbb{A} \to \mathbb{R} \to \mathbb{C} \to \mathbb{H}$

Next lecture Proofs

HomeworkNumber of primes?

Sources

- Wiles, Fermat, Four color map, Gdel, Icosahedron, Euler, BinomConvergence, Julia Set, Three apples, Cattle count, Yanghui triangle, Seki Kowa, Kronecker, Goldbach Letter: Wikipedia, Wikimedia
- Find X: popular
- Loto 6: `www.takarakuji-official.jp`
- graphs: TikX Example web site and self made
- music: TUG TEX Show Case
- Wepemnefret: `http://euler.slu.edu/escher/index.php/History_and_Numbers`