I211 Logic and Mathematics

4. Lecture

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Monday Lecture

Proofs

Important points

- $\wedge$, $\vee$, $\rightarrow$, $\neg$, $\forall$, $\exists$
- $\neg\forall n P(n) \quad \Leftrightarrow \quad \exists n \neg P(n)$
- necessary, sufficient, neccessary and sufficient
- truth tables
- direct proof method
- contraposition: $A \rightarrow B \quad \Leftrightarrow \quad \neg B \rightarrow \neg A$
- proof by contradiction

Induction

Principle of Induction

Considering a proposition $P(n)$

Show that $P(0)$ hold

and

show that for all natural numbers n, $P(n) \rightarrow P(n+1)$ holds.

Then

$P(n)$ holds for all natural numbers.

Rule of induction

$$\frac{P(0) \qquad \forall n \in \mathbb{N} : P(n) \rightarrow P(n+1)}{\forall k \in N : P(k)}$$

Example

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Conclusion 1

- write down that you use induction
- write down what is the sentence you want to prove
- prove $P(0)$
- prove $P(n) \rightarrow P(n+1)$
- qed $\square$

Example

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$$

Example

$$3|(n^3 - n)$$

Conclusion 2

One needs to use the assumption

Example

Everyone loves Funazushi!

Funazushi

Proposition $P(n)$

Within a group of n people, everyone
has
the same feeling regarding Funazushi.

Conclusion

I like Funazushi
thus everyone likes Funazusi.

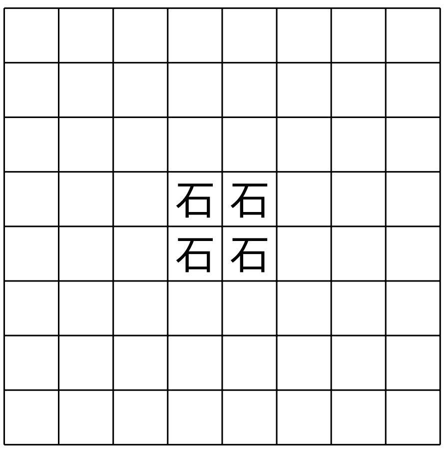
Conclusion 3

Don't get tricked! It is not about convincing each other!

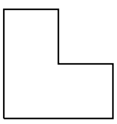
Example

The center of a stone garden

Let's plaster a $2^n \times 2^n$ stone garden with tiles



Tile shape



All $2^n \times 2^n$ stone gardens can be tiled, so that in the center one piece remains.

Conclusion 4

Sometimes it is better to use a stronger induction hypothesis!

Next lesson

More induction