L211 Logic and Mathematics

7. Lecture

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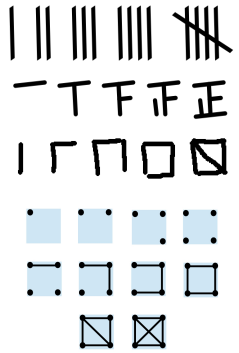
History: the first notation systems for numbers

Number systems

Tally marks



Europe, North America etc

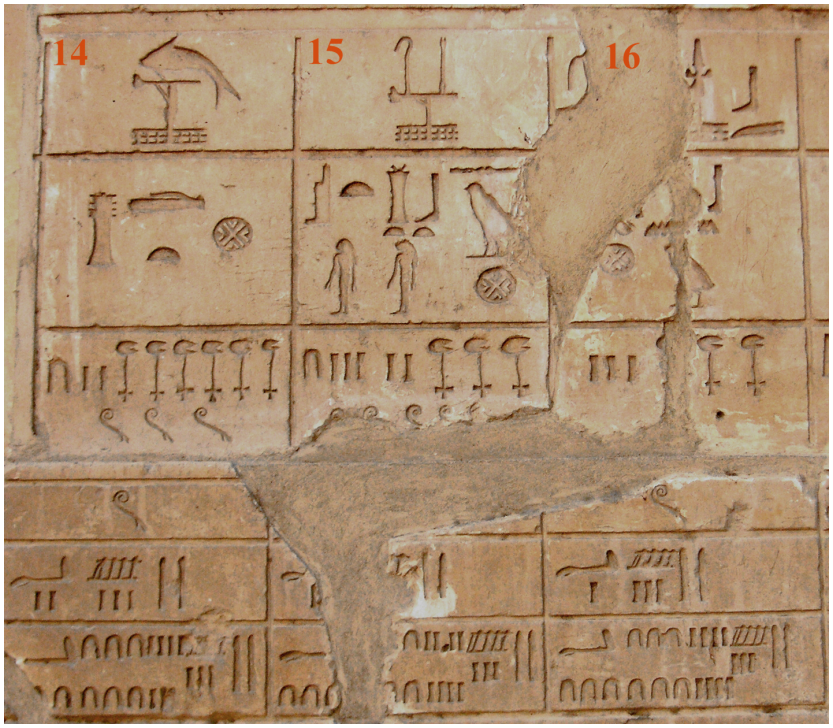
Kanji area

France, Spain, South America

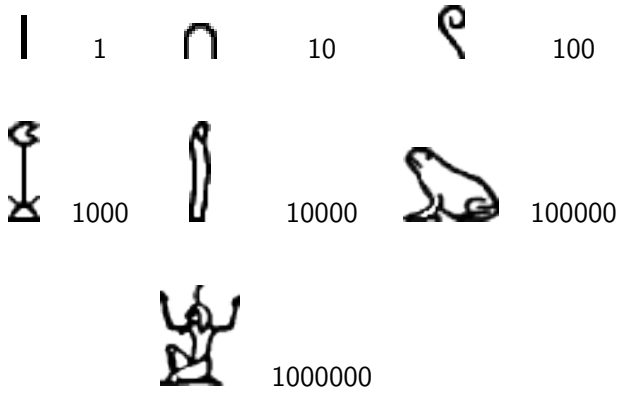
Forestry

Unary numeral system

What to do with big numbers?



Egyptian number system

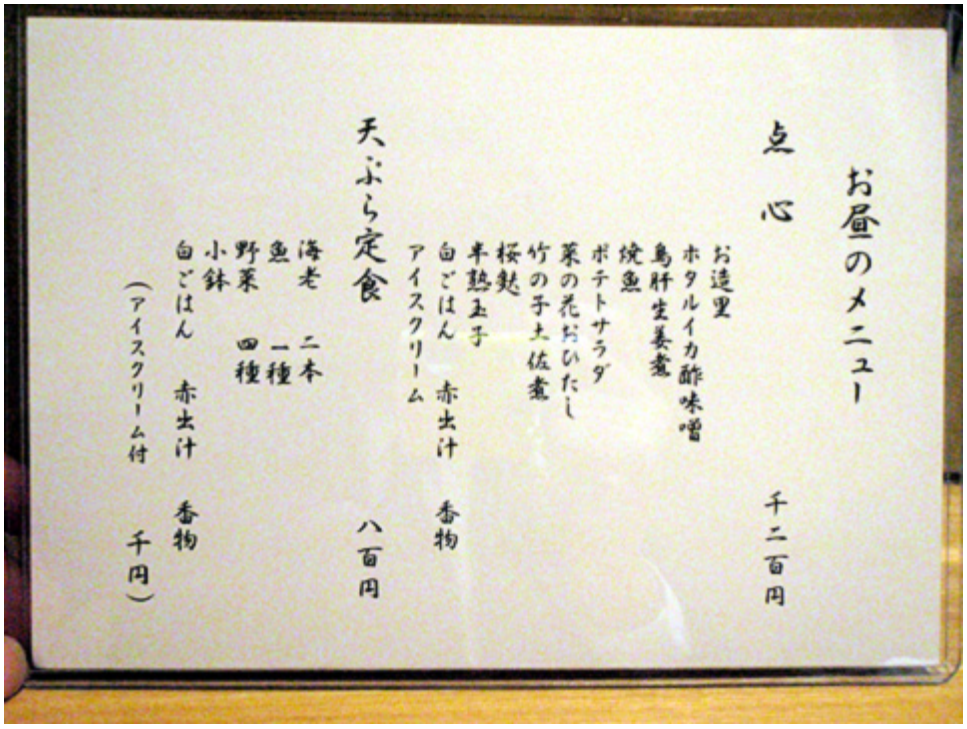


Roman number system

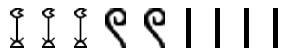
I	1	II	2	III	3	IV	4	V	5
VI	6	VII	7	VIII	8	IX	9	X	10
L	50	C	100	D	500	M	1000		

MMXV

Decimal systems



Decimal system



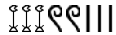
MMMCCIV

三千二百四

3204

Which one is *not* a decimal notation system? MMMCCIV !

Decimal number system and poitional notation

 $= 1000 + 1000 + 1000 + 100 + 100 + 1 + 1 + 1 + 1$

三千二百四 $= 3 \cdot 1000 + 2 \cdot 100 + 4 \cdot 1$

3203 $= 3 \cdot 1000 + 2 \cdot 100 + 0 \cdot 10 + 4 \cdot 1$

Reason of a positional notation system

$$\begin{array}{rr} & \text{三千六百四} \\ + & \text{五百七十八} \\ \hline = & \text{四千百八十二} \end{array}$$

Decimal system

$$d_n d_{n-1} \dots d_1 d_0 = \sum_{k=n}^{0} d_k 10^k$$

Example

$$42 = 4 \cdot 10^1 + 2 \cdot 10^0$$
$$305 = 3 \cdot 10^2 + 0 \cdot 10^1 + 5 \cdot 10^0$$

$$\begin{array}{rr} & \text{三千六百四} \\ + & \text{五百七十八} \\ \hline = & \text{四千百八十二} \end{array} \qquad \begin{array}{rr} & 3604 \\ + & 578 \\ \hline = & 4182 \end{array}$$

Number systems with base 2, 10, 16

10-base (decimal)

$$(d_n d_{n-1} \dots d_1 d_0)_{10} = \sum_{k=n}^{0} d_k 10^k$$

2-base (binary)

$$(b_n b_{n-1} \dots b_1 b_0)_2 = \sum_{k=n}^{0} b_k 2^k$$

16-base (hexadecimal)

$$(h_n h_{n-1} \dots h_1 h_0)_{16} = \sum_{k=n}^{0} h_k 16^k$$

How many number symbols (digits) are necessary?

Number of digits

10-base $(d_n d_{n-1} \dots d_1 d_0)_{10} = \sum_{k=n}^{0} d_k 10^k$ 3204

Digits: 0, 1, 2, ..., 9

2-base $(b_n b_{n-1} \dots b_1 b_0)_2 = \sum_{k=n}^{0} b_k 2^k$ 110010000100

Digits: 0, 1

16-base $(h_n h_{n-1} \dots h_1 h_0)_{16} = \sum_{k=n}^{0} h_k 16^k$ C84

Digits: 0, 1, 2, ..., 9, A, B, C, D, E, F

Usage of the binary system

- Morse code
- Electro-communication
- Conmputer

Addition in base 2

$$\begin{array}{rr} & 111000010100 \\ + & 1001000010 \\ \hline = & 1000001010110 \end{array}$$

Base 2 and base 16

$$(3604)_{10} = (111000010100)_2 = (E14)_{16}$$

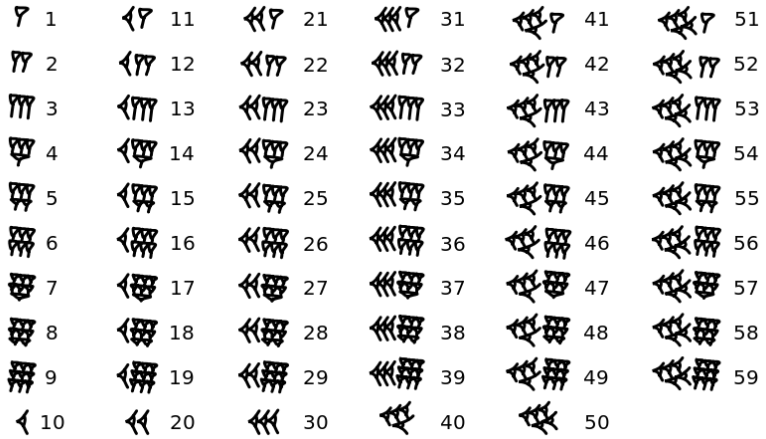
$$\begin{array}{c} 1110\,0001\,0100 \\ \underbrace{1110}\,\underbrace{0001}\,\underbrace{0100} \\ \mathsf{E}\quad 1\quad 4 \end{array}$$

Other bases?

Base 12 (duodecimal)

- Digits: 0, 1,. . . , 9, ↊, ↋
- Time: 12 hours, 12 months
- Chinese calendar
- Number of factors is big: $1/2, 1/3, 1/4, 2/3, 3/4$ all have finite decimal representation
- it is often called the best number system!

Base 60 (sexagesimal)

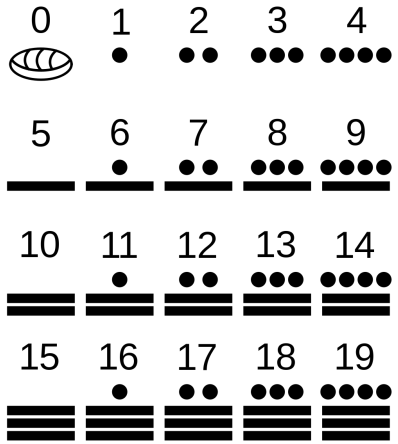


Babylonian number system

Other base 60 cases

- 1h = 60min, 1min = 60sec
- measurement of angles
- Chinese calendar (5×12)

Base 20 (vigesimal)



Maya number system

Other base 20 cases

- Ainu language
- North and South America: Maya, Aztec, Tlingit, Inuit
- Asia: Dzongkha, Santali
- French: mixed base 20/60 system:
  quatre-vingts $= 4 \cdot 20 = 80$
  soixante-quinze $= 60 + 15 = 75$
- ...

Papua New Guinea, Telefomin: base 27

Last but not least

Sources

Telefol: `http://www.number27.org/`
Menu: `http://nomchan.exblog.jp/15946226/`
Other images: Wikipedia